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Middle Years Programme

Mathematics guide

For use from January 2008 or September 2008

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IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.

IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

IB learners strive to be:

Inquirers They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy learning and this love of learning will be sustained throughout their lives.

Knowledgeable They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines.

Thinkers They exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions.

Communicators They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others.

Principled They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them.

Open-minded They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to grow from the experience.

Caring They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment.

Risk-takers They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are brave and articulate in defending their beliefs.

Balanced They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others.

Reflective They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.

Contents

How to use this guide

The *Mathematics guide* provides the framework for teaching and learning in mathematics in the Middle Years Programme (MYP) and must be read and used in conjunction with the document *MYP: From principles into practice* (August 2008).

This guide was originally published in July 2007 for use from January 2008 (southern hemisphere) and September 2008 (northern hemisphere). However, the document *MYP: From principles into practice* (August 2008) now includes all general information about the programme and, as a result, the format of subject-group guides has been changed so that they include only subject-specific information.

This revised edition of the *Mathematics guide* includes all subject-specific information as published in the earlier version. Importantly, requirements for the subject, aims, objectives and final assessment details have not changed. However, general information about the MYP has been taken out and some additional subject-specific information included (for example, sample questions related to each of the areas of interaction).

Introduction to MYP mathematics

> Mathematics, rightly viewed, possesses not only truth, but also supreme beauty.
>
> Bertrand Russell

Mathematics plays an essential role both within the school and in society. It promotes a powerful universal language, analytical reasoning and problem-solving skills that contribute to the development of logical, abstract and critical thinking. Moreover, understanding and being able to use mathematics with confidence is not only an advantage in school but also a skill for problem solving and decision-making in everyday life. Therefore, mathematics should be accessible to and be studied by all students.

Mathematics is well known as a foundation for the study of sciences, engineering and technology. However, it is also increasingly important in other areas of knowledge such as economics and other social sciences. MYP mathematics aims to equip all students with the knowledge, understanding and intellectual capabilities to address further courses in mathematics, as well as to prepare those students who will use mathematics in their workplace and life in general.

In MYP mathematics, the four main objectives support the IB learner profile, promoting the development of students who are knowledgeable, inquirers, communicators and reflective learners.

Knowledge and understanding promotes learning mathematics with understanding, allowing students to interpret results, make conjectures and use mathematical reasoning when solving problems in school and in real-life situations.

Investigating patterns supports inquiry-based learning. Through the use of investigations, teachers challenge students to experience mathematical discovery, recognize patterns and structures, describe these as relationships or general rules, and explain their reasoning using mathematical justifications and proofs.

Communication in mathematics encourages students to use the language of mathematics and its different forms of representation, to communicate their findings and reasoning effectively, both orally and in writing.

Reflection in mathematics provides an opportunity for students to reflect upon their processes and evaluate the significance of their findings in connection to real-life contexts. Reflection allows students to become aware of their strengths and the challenges they face as learners.

Overall, MYP mathematics expects all students to appreciate the beauty and usefulness of mathematics as a remarkable cultural and intellectual legacy of humankind, and as a valuable instrument for social and economic change in society.

This guide provides both MYP teachers and students with:

- the requirements of the course
- strategies to incorporate the areas of interaction into mathematics
- aims and objectives for MYP mathematics
- the prescribed curriculum framework
- details of final assessment requirements, including moderation and monitoring of assessment.

IB-produced teacher support material (TSM) is available to complement this guide and aid the implementation of the course in schools.

The IB mathematics continuum

MYP mathematics builds on experiences in mathematics learning that students have gained in their time in the IB Primary Years Programme (PYP). PYP teaching and learning experiences challenge students to be curious, ask questions, explore and interact with the environment physically, socially and intellectually to construct meaning and refine their understanding. The use of structured inquiry is a precursor to the problem-solving and inquiry-based approach of MYP mathematics. Students continuing on to the IB Diploma Programme (DP) will have developed not only an inquiring and reflective approach to mathematics learning but also critical-thinking and problem-solving skills, which they will be able to apply and extend in further DP mathematics courses. In particular, the MYP framework for mathematics reflects the concepts and skills of the presumed knowledge for the DP mathematics courses at standard level (SL) and higher level (HL). The two levels of the MYP mathematics courses (standard and extended) have been refined to allow a smooth transition from MYP mathematics to DP mathematics courses.

Aims and objectives

Aims

The aims of any MYP subject state in a general way what the teacher may expect to teach or do, and what the student may expect to experience or learn. In addition, they suggest how the student may be changed by the learning experience.

The aims of teaching and learning mathematics are to encourage and enable students to:

- recognize that mathematics permeates the world around us
- appreciate the usefulness, power and beauty of mathematics
- enjoy mathematics and develop patience and persistence when solving problems
- understand and be able to use the language, symbols and notation of mathematics
- develop mathematical curiosity and use inductive and deductive reasoning when solving problems
- become confident in using mathematics to analyse and solve problems both in school and in real-life situations
- develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics
- develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others
- develop a critical appreciation of the use of information and communication technology (ICT) in mathematics
- appreciate the international dimension of mathematics and its multicultural and historical perspectives.

Objectives

The objectives of any MYP subject state the specific targets that are set for learning in the subject. They define what the student will be able to accomplish as a result of studying the subject.

The following objectives for mathematics relate directly to the assessment criteria A–D (see "Mathematics assessment criteria").

A Knowledge and understanding

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop problem-solving skills. Through knowledge and understanding students develop mathematical reasoning to make deductions and solve problems.

At the end of the course, students should be able to:

- know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics)
- use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations, including those in real-life contexts
- select and apply general rules correctly to solve problems, including those in real-life contexts.

B Investigating patterns

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Mathematical inquiry encourages students to become risk-takers, inquirers and critical thinkers. The ability to inquire is invaluable in the MYP and contributes to lifelong learning.

Through the use of mathematical investigations, students are given the opportunity to apply mathematical knowledge and problem-solving techniques to investigate a problem, generate and/or analyse information, find relationships and patterns, describe these mathematically as general rules, and justify or prove them.

At the end of the course, when investigating problems, in both theoretical and real-life contexts, students should be able to:

- select and apply appropriate inquiry and mathematical problem-solving techniques
- recognize patterns
- describe patterns as relationships or general rules
- draw conclusions consistent with findings
- justify or prove mathematical relationships and general rules.

C Communication in mathematics

Mathematics provides a powerful and universal language. Students are expected to use mathematical language appropriately when communicating mathematical ideas, reasoning and findings—both orally and in writing.

At the end of the course, students should be able to communicate mathematical ideas, reasoning and findings by being able to:

- use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations
- use different forms of mathematical representation (formulae, diagrams, tables, charts, graphs and models)
- communicate a complete and coherent mathematical line of reasoning using different forms of representation when investigating complex problems.

Students are encouraged to choose and use ICT tools as appropriate and, where available, to enhance communication of their mathematical ideas. ICT tools can include graphic display calculators, screenshots, graphing, spreadsheets, databases, and drawing and word-processing software.

D Reflection in mathematics

MYP mathematics encourages students to reflect upon their findings and problem-solving processes. Students are encouraged to share their thinking with teachers and peers and to examine different problem-solving strategies. Critical reflection in mathematics helps students gain insight into their strengths and weaknesses as learners and to appreciate the value of errors as powerful motivators to enhance learning and understanding.

At the end of the course, students should be able to:

- explain whether their results make sense in the context of the problem
- explain the importance of their findings
- justify the degree of accuracy of their results where appropriate
- suggest improvements to the method when necessary.

Requirements

MYP mathematics is a compulsory component of the MYP in **every** year of the programme.

Organizing mathematics in the school

Schools are responsible for developing their MYP mathematics curriculum so that the final aims and objectives set by the IB can be met successfully at the end of the programme. The MYP allows schools great flexibility in the way they structure and schedule their courses so that these also meet the requirements of their local and national systems.

Teaching hours

It is essential that teachers be allowed the number of teaching hours necessary to meet the requirements of the MYP mathematics course in their particular school. Although the prescribed minimum teaching time in any given year for each subject group is 50 teaching hours, the IB recognizes that, in practice, more than 50 teaching hours per year will be necessary, not only to meet the programme requirements over the five years, but also to allow for the sustained, concurrent teaching of disciplines that enables interdisciplinary study.

Schools must ensure that students are given sufficient time and instruction to allow them the opportunity to meet the final aims and objectives for mathematics.

Framework for mathematics

MYP mathematics provides a framework of concepts and skills organized into the following five branches of mathematics.

- **Number**
- **Algebra**
- **Geometry and trigonometry**
- **Statistics and probability**
- **Discrete mathematics**

Schools are required to structure their mathematics curriculum so that the five branches, as described in the framework, are addressed over the five years (or complete duration) of the programme.

Schools are expected to use the framework for mathematics as a tool for curriculum mapping to assist them in the vertical and horizontal planning of their courses and in the development of units of work in mathematics. There is no prescription for a particular order or sequence in which the branches of the framework should be addressed, or the way in which the concepts and skills should be used when structuring units of work in mathematics. Schools are given the opportunity to develop their courses and structure their units of work to suit their own preferences and students' needs.

However, **over the five years of the programme**, schools **must** ensure that they provide students with the opportunity to experience learning in **all** the branches of the framework, ensuring that the aims and objectives for MYP mathematics are not compromised.

Levels of mathematics

MYP mathematics should be accessible to and be studied by all students. Schools must ensure that the mathematics curriculum allows **all** students the opportunity to reach their full potential and achieve the final aims and objectives of MYP mathematics. In order to achieve this, the concepts and skills of the framework for mathematics are organized so that students can work at two levels of ability: **standard mathematics** and **extended mathematics**.

Standard mathematics aims to give all students a sound knowledge of basic mathematical concepts while allowing them to develop the skills needed to meet the objectives of MYP mathematics.

Extended mathematics consists of the standard mathematics framework supplemented by additional concepts and skills. This level provides the foundation for students who wish to pursue further studies in mathematics, for example, mathematics higher level (HL) as part of the IB Diploma Programme.

IB validation of students' grades and certification are available for both standard and extended mathematics. Schools may decide to offer one or both levels, and will then allocate students to the appropriate level.

The assessment criteria for mathematics, directly addressing the aims and objectives of the course, apply to both levels. For examples of how to apply these criteria when assessing students' work, please refer to the mathematics teacher support material (TSM) that complements this guide.

Differentiated instruction and special educational needs

It is acknowledged that students do not learn mathematics at the same speed, in the same manner, or respond in the same way to the same teaching strategies. Students of the same year level may differ substantially in their mathematical abilities, as well as in their background and previous mathematical experiences. They may also have different interests and exhibit preferred ways of learning. However, it is important that all students are provided with a positive learning experience in mathematics and have the opportunity to maximize their potential.

In mixed-ability classrooms, teachers have to differentiate their instruction and adapt their assessment tasks to meet the wide range of skills and capabilities. It is the responsibility of schools and teachers to develop teaching and learning strategies that allow all students the opportunity to work towards meeting the final objectives of MYP mathematics.

There are a number of ways in which teachers can differentiate their instruction. Teachers may:

- examine the course content and determine what essential understanding is required for different students
- focus on the outcomes and allow for different ways to demonstrate understanding
- assess how space, time and resources can be best used to create effective conditions to enhance learning for all students.

For further information and support on differentiated instruction and how to create an environment that is inclusive of students with special educational needs, please refer to the SEN page, SEN resources and forums on the OCC or contact sen@ibo.org.

Resources

The resources and tasks used should be carefully chosen and prepared so that the objectives can be met and the assessment criteria can be applied. The choice of resources within a school will also need to reflect the ability range within that school.

Library

Schools should provide teachers and students with a good variety of resources to support teaching and learning in mathematics. A well-resourced and up-to-date library equipped with books, magazines and multimedia, and which reflects the ability range within the school, can contribute to sustaining students' curiosity and stimulating their interest.

Information and communication technology (ICT)

The appropriate use of computers, computer applications and calculators can improve the understanding of all students.

Depending upon the school resources, ICT should be used whenever appropriate:

- as a means of expanding students' knowledge of the world in which they live
- as a channel for developing concepts and skills
- as a powerful communication tool.

ICT provides a wide range of resources and applications for teachers to explore in order to enhance teaching and learning.

In mathematics, ICT can be used as a tool to perform complicated calculations, solve problems, draw graphs, and interpret and analyse data. ICT can also be helpful to:

- investigate data and mathematical concepts
- obtain rapid feedback when testing out solutions
- observe patterns and make generalizations
- move between analytical and graphical representation
- visualize geometrical transformations.

In addition, the appropriate use of ICT can enhance students' communication skills, assisting them in the collection, organization and analysis of information and in the presentation of their findings.

However, for ICT to be a useful tool for learning, students need to be familiar with the resources and applications, and know how and when to use them. Students should be able to decide when the use of ICT is appropriate and when alternative methods such as pencil and paper, mental calculation, or diagrams should be used. Therefore, it is important that teachers show students how to use these resources effectively while supporting the development of their intellectual skills.

ICT can support students with special educational needs who have difficulties understanding a particular concept or who would benefit from further practice. It can also provide the extra challenge for gifted and talented students to explore further ideas and concepts. "Adaptive technologies" can enable students with severe learning disabilities to become active learners in the classroom alongside their peers. For more information about adaptive technologies and special educational needs, please refer to the SEN page on the OCC.

Depending on the school facilities and availability of ICT resources, teachers are encouraged to use ICT whenever possible and appropriate as a means of enhancing learning.

Some of the possible ICT resources in mathematics might include:

- databases and spreadsheets
- graph plotter software
- dynamic geometry software
- computer algebra systems

- programming languages
- mathematics content-specific software
- graphic display calculators (GDC)
- Internet search engines
- CD-Roms
- word processing or desktop publishing
- graphic organizers.

Language of instruction

In schools where the language of instruction of mathematics is not the mother tongue of some of the students taking the course, measures must be implemented to ensure that these students are not disadvantaged and have the full opportunity to demonstrate the highest achievement level in the final objectives. These measures may include:

- teacher training
- modification of language in materials
- differentiation of assessment tasks
- parallel resources in students' mother tongues.

For further information, please refer to the document *Learning in a language other than mother tongue in IB programmes*.

Professional development

To support teachers in meeting the aims and objectives of MYP mathematics, professional development must be carefully planned within the school. Opportunities to attend in-school workshops and IB regional conferences should be provided, to ensure that teachers develop a good understanding of the underpinning philosophy of the MYP and of the requirements of MYP mathematics in particular.

The online curriculum centre (OCC)

The OCC is a valuable resource for teachers in the MYP. Teachers are encouraged to participate in and contribute to this resource as a means of developing the IB online learning community. The OCC contains discussion forums and resource banks for all MYP subject groups, the personal project, special educational needs and academic honesty.

IB-appointed faculty members answer queries and provide advice on teaching and learning, implementation and moderation. Teachers can post queries, share resources and download all IB official publications. Please see your MYP coordinator for a school code and password.

Developing the curriculum

Introduction

All MYP subjects, including mathematics, provide a curricular framework with set final aims and objectives. Schools are responsible for developing and structuring their mathematics courses so that they provide opportunities for students to meet the final aims and objectives effectively by the end of the programme.

Teachers are expected to map the teaching and learning experiences that students will encounter as they move from one year to the next in the programme. The MYP mathematics courses should be carefully sequenced and articulated so that they contribute to the development of students' conceptual understanding, practical and intellectual skills as well as personal beliefs and values.

The MYP requires schools to facilitate and promote collaborative planning for the purpose of curriculum planning, review and reflection.

The staff responsible for teaching and learning in mathematics will need to determine the subject content for each year of the programme to make sure the five branches of the framework are covered over the five years (or complete duration) of the programme. All objectives must be developed in each year of the programme, at the appropriate level. In planning the mathematics curriculum, teachers will need to deconstruct the objectives so that they build during years 1–4 towards the highest level in the final year of the programme, providing for continuity and progression in each objective. The objectives in this guide, and the examples of interim objectives for mathematics available on the OCC, will guide teachers in making decisions about the choice of content and learning experiences offered to students, including the types of assessment that are appropriate for the students' particular stages of development.

In developing the curriculum for the different years of the programme, teachers are encouraged to plan increasingly complex tasks or units of work that will cover the entire scope of the objectives themselves. However, within these, discrete tasks or smaller units of work might concentrate on specific objectives.

In the final year of the programme, the curriculum should provide students with the **opportunity** to achieve the highest descriptor levels in the final assessment criteria (see "Mathematics assessment criteria").

The document *MYP: From principles into practice* (August 2008) provides detailed information on organizing the written, assessed and taught curriculum, including the use of interim objectives, modified assessment criteria for years 1–4 of the programme, and the planning of units of work.

Developing the curriculum within the subject

While having to meet national requirements and local standards, teachers should ensure that the curriculum they develop reflects the principles and practice of the MYP. The fundamental concepts and the IB learner profile should act as guiding principles when developing the curriculum in the school.

Teaching and learning strategies

In order to give all students opportunities to meet the MYP mathematics objectives, teachers should provide classroom environments that enhance learning and use a range of teaching and learning strategies to challenge all students.

To achieve this, MYP teachers should adopt the following strategies.

Use the areas of interaction as starting points for teaching and learning

Teaching mathematics through the areas of interaction enhances the learning experience in mathematics. The use of the areas of interaction introduces a new dimension to the inquiry and allows for a richer and in-depth exploration of concepts and topics. The areas of interaction can be used as starting points to develop units of work in mathematics, or as bridges to explore connections with other disciplines and real-world issues.

Allow students to communicate their mathematical thinking

Reading and interpreting mathematics texts, problems, functions and equations does not come naturally to most students. Some words and symbols have different meanings in mathematics and in everyday use. Many students also access the curriculum in a language other than their mother tongue. Students need to become familiar with the language of mathematics in order to communicate their ideas and findings with increasing confidence.

Teachers can help students understand the language of mathematics and master the skills of communication by providing them with tasks that allow them to read mathematics texts, to express their lines of reasoning and to communicate their findings using the appropriate mathematical language (terminology, notation, symbols) and format.

Teachers can assist students' comprehension by rephrasing instructions, speaking problems aloud and explaining their reasoning so that students learn and carry out mathematical tasks with understanding.

Devise investigations to explore mathematical concepts and ideas

MYP mathematics expects teachers to devise investigations where students choose their own strategies and methods while attempting to solve problems. Investigations can involve real-life situations or purely mathematical ones. MYP mathematics emphasizes open-ended investigations where more than one answer is possible.

Use real-life contexts and situations

When students solve problems that have been framed in real-life contexts or that are relevant to their interests, they make connections between what they learn in the classroom and its applications to other subjects and the real world. Connecting mathematical ideas and concepts to other subjects and real-life contexts enhances the understanding that learning mathematics is meaningful and functional. This allows students to reason and use mathematics when solving problems in mathematics and in other contexts.

In general, good practice in mathematics teaching is changing. Some teaching practices that have become more effective for increasing students' understanding of mathematics are listed in the table below. These changes should be reflected in the MYP classroom.

How is mathematics teaching changing?	
Increased emphasis on:	**Decreased emphasis on:**
connecting mathematical concepts and applications	treating mathematics as isolated concepts and facts
developing mathematical understanding through the development of reasoning and analytical skills, making mathematics more meaningful to students	rote practice, memorization and symbol manipulation
solving real-life problems in which the context is relevant to the student	word problems as problem solving

How is mathematics teaching changing?	
Increased emphasis on:	**Decreased emphasis on:**
instruction that builds on what students know and need to learn	instruction focused on what students do not know
a variety of strategies for possible multiple solutions	one method, one answer
students being encouraged to speculate and pursue ideas	the teacher as the sole authority for providing the right answers
explaining processes in a clear and logical way and reflecting upon results	finding answers
teachers working in teams with colleagues from their own and other subject groups	teachers working in isolation
multiple sources and resources for learning	a textbook-driven curriculum
students investigating, questioning, discussing and justifying or proving	the use of exercise sheets
practical activities, including groups or collaborative tasks according to the activity	a "chalk and talk" lesson format
assessment as an integral part of instruction (formative assessment)	final examinations
a broad range of assessment strategies, including tests where students have to show their reasoning.	short-answer, multiple-choice assessment.

Developing units of work

When planning a unit of work in mathematics, teachers should ensure that:

- relevant aspects of the unit of work are presented through the perspective of at least one of the areas of interaction
- mathematical knowledge, understanding and skills are being developed
- interdisciplinary teaching is explored and used where appropriate
- differentiated instruction and diverse teaching strategies are used to cater for inquiry-based learning and multiple levels of ability
- real-life situations are used as the context for mathematics tasks, where appropriate
- local and/or global issues are used to promote inquiry into the role of mathematics in society and the environment
- tasks allow students to think about the problem-solving processes, reflect upon their methods and results, and explore the connection with everyday life
- assessment tools such as assessment rubrics, with clear descriptions of assessment outcomes, are shared with all students and these outcomes reflect the MYP mathematics aims and objectives (see "Aims and objectives")
- learner outcomes match the MYP objectives (see objectives in "Aims and objectives") and are considered throughout the five years of the programme
- student achievement of the objectives is measured against the assessment criteria (see "Mathematics assessment criteria").

Addressing the areas of interaction

The areas of interaction provide contexts through which teachers and students consider teaching and learning, approach the disciplines, and establish connections across disciplines. They are organizing elements that strengthen and extend student awareness and understanding through meaningful exploration of real-life issues. All teachers share the responsibility of using the areas of interaction as a focus for their units of work.

The process of inquiring into the subject content through the different perspectives or contexts of the areas of interaction enables students to develop a deeper understanding of the subject as well as the dimensions of the areas of interaction. Through this inquiry cycle of understanding and awareness, reflection and action, students engage in reflection and metacognition, which can lead them from academic knowledge to thoughtful action, helping to develop positive attitudes and a sense of personal and social responsibility.

The document *MYP: From principles into practice* (August 2008), in the section "The areas of interaction", provides further information relating to the dimensions of each area of interaction, the inquiry cycle, planning units of work, and focusing relevant content through these areas of interaction.

There are five areas of interaction:

- approaches to learning (ATL)
- community and service
- health and social education
- environments
- human ingenuity (formerly *homo faber*).

The following sections on the areas of interaction provide sample questions that might be used as **MYP unit questions** or **inquiry cycle questions**, depending on the content being taught. These particular questions are "content free", and when devising their own questions, teachers can relate them to the specific content that is being explored in a unit of work.

It is important to note that the areas of interaction are ways of looking at content: some of the examples that follow could easily fit into more than one area of interaction perspective, and also have the potential to be explored through subjects other than mathematics.

The contexts that frame the content curriculum in mathematics must be natural and meaningful. Often when designing a unit of work, the context for the content will emerge naturally. To provide meaningful learning experiences, teachers should ensure that the MYP unit question gives students scope for inquiry into the issues and themes within the content. The area of interaction will then give direction to teacher-directed and student-initiated inquiry.

Please note that any reference to "I" in the areas of interaction questions could also be interpreted as "we" where this is more appropriate to the social ethos of the school or location.

Approaches to learning

How do I learn best?
How do I know?
How do I communicate my understanding?

Approaches to learning (ATL) are central to all MYP subject groups and the personal project. Through ATL, schools provide students with the tools to enable them to take responsibility for their own learning. This involves articulating, organizing and teaching the skills, attitudes and practices that students require to become successful learners.

The MYP has identified seven groups of skills that encompass ATL: organization, collaboration, communication, information literacy, reflection, thinking, and transfer. The school community will need to spend time defining the ATL attitudes, skills and practices that they consider important within these groups, both for an individual subject group and across subject groups.

Some activities that could be used to develop ATL skills through mathematics include:

- using deductive reasoning to formulate a hypothesis that could be scientifically tested by an experiment
- analysing and interpreting patterns and information presented in tables, charts and graphs from various resources such as newspapers, magazines and other publications
- using open-ended investigations that have more than one possible solution and allow for more than one possible problem-solving strategy to encourage divergent thinking
- using Escher tessellations to examine geometry and design principles, and exploring how mathematics can be used to create artistic designs and effects
- using national lotteries and other gambling games to gain insight into probabilities and the chances of winning
- using real-life problems such as traffic jams, queues in the supermarket or games situations to design mathematical models based on probabilities and plan solutions to these problems
- using networks and flow diagrams as tools for making decisions for planning a travel itinerary
- using the concept of algorithm for planning and scheduling tasks for the personal project.

Sample questions

- What do we learn in mathematics? How is learning in mathematics similar to or different from learning in other subjects?
- What skills are specific to mathematics?
- How is communication in mathematics different from other subjects?
- How does learning mathematics help me with learning in other subjects?
- What skills and knowledge can I take from other subjects and use in my mathematics learning?
- How can I plan and organize my learning more effectively?
- What are effective ways of working with my classmates? How can collaborative work improve my mathematics skills?
- What is the value of reflection in mathematics?
- How can information and communication technology (ICT) help my mathematics learning?
- How can learning mathematics improve my thinking skills?

Students may be aiming to address these questions in mathematics by the end of the programme.

Community and service

How do we live in relation to each other?
How can I contribute to the community?
How can I help others?

The emphasis of community and service is on developing community awareness and a sense of belonging and responsibility towards the community so that students become engaged with, and feel empowered to act in response to, the needs of others.

Community and service starts in the classroom and extends beyond it, requiring students to discover the social reality of self, others and communities. This, in turn, may initiate involvement and service in the communities in which they live. Reflection on the needs of others and the development of students' ability to participate in and respond to these needs both contribute to the development of caring and responsible learners.

Students will explore the nature of past and present communities through mathematics, as well as their place in their own communities. Incorporating community and service into the study of mathematics encourages responsible citizenship as students deepen their knowledge and understanding of the world around them.

Activities that may be considered to integrate community and service through mathematics include:

- organizing a fund-raising event in the school to raise money for a charity; preparing a simple budget, estimating expenses, incomes and profit for the various activities
- using tests to measure the fitness of different groups of a community; analysing results, considering age, activity, smoking habits, and so on; communicating the results using comparative tables and graphs, and developing posters to raise awareness of the importance of fitness for a healthy society
- using local newspapers to analyse articles related to statistics and social issues, and discussing how statistics can inform as well as mislead
- using a local road safety leaflet to explore concepts of speed, acceleration, distance and displacement; producing leaflets for the community to raise awareness of the importance of reducing speed around school areas.

Sample questions

- What is the role of mathematics in a community and in the world?
- How is the knowledge of mathematics useful in communities?
- How can a community influence the learning of mathematics?
- What is my role in the community? How can I contribute to my community through mathematics?
- What would the world be like without mathematics?
- How can I improve my community through what I've learned in mathematics?

Students may be aiming to address these questions in mathematics by the end of the programme.

Health and social education

How do I think and act?
How am I changing?
How can I look after myself and others?

This area of interaction is about how humanity is affected by a range of social issues (including health). It includes an appreciation of these effects in various cultural settings and at different times. It is concerned with physical, social and emotional health and intelligence—key aspects of development leading to a complete and balanced lifestyle.

Activities that may be considered to integrate health and social education in mathematics include:

- investigating proportions and ratios—looking at food dishes from different cultures and performing calculations for scaling up recipes for the whole class
- using observations—investigating traffic through observations, as well as data analysis and statistics, to promote a road safety campaign around the school

- investigating population growth—using data analysis, statistics and probability to compare growth rates of different countries
- investigating the applications of bar codes, the process of encrypting and decrypting data, or the flow of traffic through one-way streets using discrete mathematics
- using mathematical functions to predict the spread of a disease or the behaviour of a population
- discussing the role of statistics and probability for providing information, its power and reliability.

Sample questions

- How does mathematics impact on society? On individuals? On me?
- Can mathematics be used to influence the health of a society?
- To what extent can mathematics contribute to the well-being of people and societies?
- How can mathematics help to communicate the health of a society and/or nation?
- In what ways does mathematics allow me to express myself?
- How does mathematics enable me to learn about myself and others?
- How can my learning in mathematics help me to make healthy choices?

Students may be aiming to address these questions in mathematics by the end of the programme.

Environments

What are our environments?
What resources do we have or need?
What are my responsibilities?

This area of interaction considers environments to mean the totality of conditions surrounding us, natural, built and virtual. It focuses on the wider place of human beings in the world and how we create and affect our environments. It encourages students to question, to develop positive and responsible attitudes, and to gain the motivation, skills and commitment to contribute to their environments.

Activities that may be considered to promote environmental awareness, responsibility, action and reflection in mathematics include:

- investigating natural resources—using techniques for measuring and analysing data to formulate questions and make predictions about the use and availability of a given resource at a given time in the future
- developing practical projects—using geometry and trigonometry to respond to the specific needs of local environments (town planning, designing and making models of real or imagined buildings, or other space-management applications)
- estimating water use—using mathematics to estimate water consumption and put forward plans for its management and conservation both at school and at home
- investigating household or school waste—predicting community waste to devise a campaign to raise awareness on waste management by suggesting ways of reducing, reusing or recycling waste
- investigating endangered species—using statistics and probability to estimate the population size of a given species at a given time, to raise awareness for the conservation and protection of species in their natural environments
- investigating climate change—using data analysis and probability to discuss the evidence of the correlation between the emission of greenhouse gases and the rise in global temperature.

Sample questions

- In what way can mathematics influence natural, built and virtual environments?
- How does mathematics influence the school environment?
- What issues do natural, built and virtual environments present for mathematics?
- How can mathematics affect our understanding of different environments?
- How do my mathematics skills enable me to understand different environments?
- How can my mathematics skills help me to improve my environments?
- What power can mathematics give us to communicate environmental issues to the world?

Students may be aiming to address these questions in mathematics by the end of the programme.

Human ingenuity (formerly *homo faber*)

Why and how do we create?
What are the consequences?

Human ingenuity looks at human contributions in the world both in their particular context and as part of a continuing process. It stresses the way humans can initiate change, whether for good or bad, and examines the consequences (intended and unintended). This area also emphasizes both the importance of researching the developments made by people across place, time and cultures, and the importance of taking time to reflect on these developments.

Mathematics is one of the greatest intellectual achievements of humankind and, as such, it provides many opportunities to incorporate human ingenuity into the curriculum. Some possible activities include:

- investigating the history and evolution of mathematics across cultures and its impact on individuals and societies
- researching mathematical inventions, for example, the universality of mathematics as a language, the invention of the concept of zero
- exploring the application of mathematical knowledge in industrial and technological developments
- developing mathematical models and formulae to describe real-life phenomena
- developing mathematical ideas, such as symmetry in geometry, through the creation of tessellations
- exploring mathematical projects with interdisciplinary themes, for example, investigating the links between geometry and art/architecture, or shapes and motifs of different cultures.

Sample questions

- What is mathematics? Where does it come from?
- How has mathematics evolved over time?
- How can mathematics initiate change?
- What would the world be like without mathematics?
- In what ways have humans shaped mathematics? In what ways has mathematics shaped our lives?
- How does mathematics affect language?
- In what ways has knowledge influenced mathematics?
- In what ways has mathematics influenced knowledge?
- How is mathematics developing in my time and culture?
- How useful is mathematics in my life?
- What contributions has mathematics made to human civilizations?

Students may be aiming to address these questions in mathematics by the end of the programme.

Framework for mathematics

The framework for MYP mathematics outlines five branches of mathematical study.

- **Number**
- **Algebra**
- **Geometry and trigonometry**
- **Statistics and probability**
- **Discrete mathematics**

Schools can use the framework for mathematics as a tool for curriculum mapping when designing and planning their mathematics courses. Schools are not expected to address all the branches of the framework in each year of the programme, nor are they asked to cover every concept and skill suggested in the framework. However, it is important that, over the five years (or complete duration) of the programme, students experience learning in all five branches of the framework for mathematics.

In this section, concepts and skills for each of the five branches of the framework for mathematics are suggested. The concepts and skills are **examples** of what students may expect to study at the two levels: standard mathematics and extended mathematics.

Concepts and skills

Number

The ability to work with numbers is an essential skill in mathematics. Students are expected to have an understanding of number concepts and to develop the skills of calculation and estimation. Students should understand that the use of numbers to express patterns and to describe real-life situations goes back to humankind's earliest beginnings, and that mathematics has multicultural roots.

Concepts	Skills
Standard and extended mathematics	
Forms of numbers: integers, fractions, decimals, exponents, standard form (scientific notation) and surds/radicals Number systems: set of positive integers and zero ($\mathbb{N}$), integers ($\mathbb{Z}$), rationals ($\mathbb{Q}$), irrationals ($\mathbb{Q}'$) and real numbers ($\mathbb{R}$)	Ordering numbers Transformation between different forms of numbers Simplification of numerical expressions in the number systems and forms of number Recognizing and classifying numbers in different number systems
The four number operations	Using the four number operations (addition, subtraction, multiplication and division) with integers, decimals and simple fractions
Prime numbers and factors, including greatest common divisor and least common multiple	Representing a number as the product of its prime factors and using this representation to find the greatest common divisor and least common multiple
Number lines	Expressing the solution set of a linear inequality on the number line
Estimation	Using different forms of rounding, decimal approximation and significant figures Using appropriate forms of rounding to estimate results
Units of measurement	Converting between different units of measurement and between different currencies
Ratio, percentage, direct and inverse proportion	Dividing a quantity in a given ratio Finding a constant of proportionality, setting up equations and graphing direct and inverse relationships
Number sequences	Predicting the next term in the number sequence (linear, quadratic, triangular, Fibonacci)
Extended mathematics only	
Fractional exponents	Using the rules of indices to simplify numerical expressions involving radicals and exponents
Absolute and percentage error in estimations	Calculating the absolute and percentage error

Algebra

Algebra is an abstraction of the concepts first used when dealing with number and is essential for further learning in mathematics. Algebra uses letters and symbols to represent number, quantity and operations, and employs variables to solve mathematical problems.

Students who wish to continue studying mathematics beyond the MYP will require knowledge of concepts and skills in algebra. Teachers should, where appropriate, assist students' understanding of algebra by using real-life contexts for the application of algebraic knowledge and skills in problem-solving situations.

Concepts	Skills
Standard and extended mathematics	
Addition, subtraction, multiplication and division of algebraic terms	Expanding and simplifying algebraic expressions
Factorization of linear and quadratic expressions	Factorizing algebraic expressions
Substitution	Using substitution to evaluate expressions
Rearranging algebraic expressions	Changing the subject of the formula
Algebraic fractions	Solving equations involving algebraic fractions
Integer exponents (including negative number exponents)	Using the laws of exponents
Patterns and sequences	Finding and justifying or proving general rules/ formulae for sequences
Functions: • types of functions—linear, quadratic • domain and range	Solving the linear function $f(x) = mx + c$, its graph, gradient and y-intercept Graphing different types of functions and understanding their characteristics Determining the range, given the domain
Graphs	Sketching and interpreting graphs
Equations: • linear • simultaneous • quadratic	Solving equations algebraically and using graphs
Inequalities	Solving and graphing linear inequalities

Concepts	Skills
Extended mathematics only	
Logarithms and exponents: • fractional exponents • logarithms with different base numbers (including natural logarithms)	Using the laws of logarithms
Functions and graphs: • types of functions: trigonometric, exponential, logarithmic, reciprocal function ($f(x)=\frac{1}{x}$ and their transformations), the square root function • inverse and composite function	Graphing different types of functions and understanding their characteristics Determining inverse and composite functions and their graphs
Equations involving the functions above	Solving equations algebraically and using graphs
Inequalities	Solving non-linear inequalities Linear programming
Arithmetic and geometric series	Developing and justifying or proving general rules/ formulae for sequences Finding the sum of the series Finding unknowns (ratio, term, and so on)
Matrices	Performing basic operations with matrices Using matrices (solving equations, transformations, growth models, and so on)

Geometry and trigonometry

The study of geometry and trigonometry enhances students' spatial awareness and provides them with the tools for analysing, measuring and transforming geometric quantities in two and three dimensions.

Concepts	Skills
Standard and extended mathematics	
Geometrical elements and their classification	Naming and classifying different geometrical elements (point, line, angle, regular and irregular planar figures, solids)
Distance	Measuring distance (between two points, between a line and a point)
Angle properties	Solving problems using the properties of: • angles in different figures or positions • acute, right and obtuse angles in triangles • angles in intersecting and parallel lines • angles in regular and irregular polygons • angles in circles
Triangle properties	Solving problems involving triangles by using: • Pythagoras' theorem and its converse • properties of similar triangles • properties of congruent triangles
Perimeter/area/volume	Finding the perimeter (circumference), area and volume of regular and irregular two-dimensional (2D) and three-dimensional (3D) shapes
The Cartesian plane	Identifying the different components of the Cartesian plane: axes, origin, coordinates (x , y) and points Understanding and using the Cartesian plane, plot graphs and measuring distances between points
Trigonometric ratios in right-angled triangles	Relating angles and sides of right-angled triangles using sines, cosines and tangents Solving problems in right-angled triangles using trigonometric ratios
Constructions	Using geometry tools to make basic constructions and using these in solving problems
Simple isometric transformation	Transforming a figure by rotation, reflection, translation and enlarging
Loci	Using the concept of locus to solve problems in two dimensions

Concepts	Skills
Extended mathematics only	
Vectors and vector spaces	Adding, subtracting and scalar multiplication of vectors
Similarity and congruence theorems	Justifying or proving theorems for congruence, similarity, shape and angles
Trigonometric ratios for angles bigger than 90°	Justifying or proving simple trigonometric identities to simplify and solve equations where $0° \leq \theta \leq 36°$
Sine and cosine rules	Using the sine and cosine rules to solve problems

Statistics and probability

This branch of mathematics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses and predict the occurrence of events.

Through the study of statistics, students should develop skills associated with the collection, organization and analysis of data, enabling them to present information clearly and to discover patterns. Students will also develop critical-thinking skills, enabling them to differentiate between what happens in theory (probability) and what is observed (statistics).

Students should understand both the power and limitations of statistics, becoming aware of their legitimate use in supporting and questioning hypotheses, but also recognizing how statistics can be used to mislead as well as to counter opinions and propaganda.

Students should use these skills in their investigations and are encouraged to use information and communication technology (ICT) whenever appropriate.

Concepts	Skills
Standard and extended mathematics	
Graphical analysis and representation (pie charts, histograms, line graphs)	Constructing and interpreting graphs
Population sampling	Selecting samples and making inferences about populations
Measures of central tendency/location (mean, mode, median, quartile, percentile)	Calculating the mean, median and mode, and choosing the best measure of central tendency
Measures of dispersion (range, inter-quartile range)	Calculating quartiles and percentiles and discussing their uses
Probability of an event Probability of exclusive and combined events Probability of successive trials	Calculating probabilities of simple events Calculating probabilities of mutually exclusive events and combined events Using tree diagrams to determine the probability of repeated events
Extended mathematics only	
Normal distribution and standard deviation	Making inferences about normal distributed data given the mean and the standard deviation
Linear regression Correlation	Drawing the line of best fit
Conditional probability	Calculating conditional probability

Discrete mathematics

Discrete mathematics is a relatively new branch of mathematics that has its roots in abstract algebra and has adopted the language and notation of graph theory. Discrete mathematics contributes to the understanding of systems and formal structures. Its techniques have become increasingly important for people to analyse and solve problems in technology, science, engineering, business and other complex systems.

Students should develop logical-thinking skills and be able to articulate their understanding through the use of Venn diagrams, structure diagrams and flow charts. Discrete mathematics provides new approaches to learning (ATL) in the MYP.

Students should be aware of the real-world applications of discrete mathematics, which may include road or rail networks, computer networks, communications networks, optimal routes, time- and project-management techniques, and critical path analysis.

Concepts	Skills
Standard and extended mathematics	
Sets Venn diagrams	Performing operations Drawing and interpreting Venn diagrams Using Venn diagrams to solve problems in real-life contexts
Logic	Expressing ideas in two-value systems (Boolean algebra) Applying truth tables to determine the truth for complex statements
Networks (including trees)	Locating paths and tours Analysing networks to find complete paths, shortest distance paths Solving problems involving optimal solutions Devising and describing procedures for performing complete calculations Using networks and flow charts to solve problems in real-life contexts
Algorithms	Analysing and using well-defined procedures for solving complex problems
Extended mathematics only	
Topology	Classifying and describing topological objects and simplifying knots
Directed networks	Performing critical path analyses
Codes and ciphers	Encoding and decoding information using translations and modular arithmetic Generating and translating bar codes Encoding and decoding with RSA codes that involve public and private keys

Assessment in the MYP

There is no external assessment provided by the IB for the MYP and therefore no formal externally set or marked examinations. All assessment in the MYP is carried out by teachers in participating schools and relies on their professional expertise in making qualitative judgments, as they do every day in the classroom. In line with the general IB assessment philosophy, a norm-referenced approach to assessment is not appropriate to the MYP. Instead, MYP schools must follow a criterion-related approach. This means that students' work must be assessed against defined assessment criteria and not against the work of other students.

The IB moderation and monitoring of assessment procedures ensure that the final judgments made by these teachers all conform to an agreed scale of measurement on common criteria.

It is expected that the procedures for assessment and the MYP assessment criteria are shared with both students and parents as an aid to the learning process.

Using the assessment criteria

The assessment criteria published in this guide correspond to the objectives of this subject group. The achievement levels described have been written with year 5 final assessment in mind.

All schools **must** use the assessment criteria published in this guide for final assessment, although local or national requirements may involve other assessment models and criteria as well.

In years 1–4, schools may modify the descriptors of the achievement levels for each criterion according to the progression of learning organized by them and guided by the interim objectives. These modified criteria must be based on the MYP principles of assessment and must provide for a coherent approach to assessment practices over the entire programme. Schools may add other criteria, in addition to the MYP criteria, in response to national requirements and report on these internally to parents and students.

Clarifying published criteria in year 5

During the final year of the programme, the final assessment criteria as published in each subject-group guide must be used when awarding levels. However, specific expectations of students for a given task must still be defined.

Teachers will need to clarify the expectations of any given task with direct reference to the published assessment criteria. For example, in mathematics, teachers would need to clarify exactly what "appropriate deductions" means in the context of a given assessment task. This might be in the form of:

- a task-specific clarification of the criteria, using the published criteria but with some wording changed to match the task
- an oral discussion of the expectations
- a task sheet that explains the expectations.

It is important that teachers specify the expected outcomes at the beginning of each individual task so that students are aware of what is required.

When clarifying expectations, teachers must ensure that they do not alter the standard expected in the published criteria, nor introduce new aspects. When awarding levels in year 5, teachers themselves should always use the published criteria.

Please also see the "Mathematics: Moderation" section for guidance on what is required as part of background information.

The "best-fit" approach

The descriptors for each criterion are hierarchical. When assessing a student's work, teachers should read the descriptors (starting with level 0) until they reach a descriptor that describes an achievement level that the work being assessed has **not** attained. The work is therefore best described by the preceding descriptor.

Where it is not clearly evident which level descriptor should apply, teachers must use their judgment to select the descriptor that best matches the student's work overall. The "best-fit" approach allows teachers to select the achievement level that best describes the piece of work being assessed.

If the work is a strong example of achievement in a band, the teacher should give it the higher achievement level in the band. If the work is a weak example of achievement in that band, the teacher should give it the lower achievement level in the band.

Further guidance

Only whole numbers should be recorded; partial levels, fractions and decimals are not acceptable.

The levels attributed to the descriptors must not be considered as fixed percentages, nor should it be assumed that there are arithmetical relationships between descriptors. For example, a level 4 performance is not necessarily twice as good as a level 2 performance.

Teachers should not think in terms of a pass or fail boundary for each criterion, or make comparisons with, or conversions to, the IB 1–7 grade scale, but should concentrate on identifying the appropriate descriptor for each assessment criterion.

The highest descriptors do not imply faultless performance, but should be achievable by students at the end of the programme. Teachers should therefore not hesitate to use the highest and lowest levels if they are appropriate descriptors for the work being assessed.

A student who attains a high achievement level for one criterion will not necessarily reach high achievement levels for the other criteria. Similarly, a student who attains a low achievement level for one criterion will not necessarily attain low achievement levels for the other criteria.

Teachers should not assume that the results of a group of students being assessed will follow any particular distribution plan.

Further information on MYP assessment can be found in the document *MYP: From principles into practice* (August 2008), in the section "Assessment".

Mathematics assessment criteria

Please note that the assessment criteria in this guide are for first use in **final assessment** in 2008 for southern hemisphere schools and 2009 for northern hemisphere schools.

The following assessment criteria have been established by the IB for mathematics in the MYP. All final assessment in the final year of the MYP must be based on these assessment criteria even if schools are not registering students for IB-validated grades and certification.

Criterion A	Knowledge and understanding	Maximum 8
Criterion B	Investigating patterns	Maximum 8
Criterion C	Communication in mathematics	Maximum 6
Criterion D	Reflection in mathematics	Maximum 6

- For each assessment criterion, a number of band descriptors are defined. These describe a range of achievement levels, with the lowest represented as 0.
- The criteria are not equally weighted.
- The descriptors concentrate on positive achievement, although failure to achieve may be included in the description for the lower levels.

Detailed descriptions of the assessment criteria and band descriptors follow.

Criterion A: Knowledge and understanding

Maximum: 8

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop skills. This criterion expects students to use their knowledge and to demonstrate their understanding of the concepts and skills of the prescribed framework in order to make deductions and solve problems in different situations, including those in real-life contexts.

This criterion examines to what extent the student is able to:

- know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics)
- use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations, including those in real-life contexts
- select and apply general rules correctly to solve problems, including those in real-life contexts.

Assessment tasks for this criterion are likely to be class tests, examinations, real-life problems and investigations that may have a variety of solutions.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors given below.
1–2	The student **attempts** to make deductions when solving **simple** problems in **familiar** contexts.
3–4	The student **sometimes** makes **appropriate** deductions when solving **simple and more-complex** problems in **familiar** contexts.
5–6	The student **generally** makes **appropriate** deductions when solving **challenging** problems in a **variety** of **familiar** contexts.
7–8	The student **consistently** makes **appropriate** deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar** situations.

Notes

1. Context: the situation and the parameters given to a problem.
2. Unfamiliar situation: challenging questions or instructions set in a new context in which students are required to apply knowledge and/or skills they have been taught.
3. Deduction: reasoning from the general to the particular/specific.

Criterion B: Investigating patterns

Maximum: 8

Students are expected to investigate a problem by applying mathematical problem-solving techniques, to find patterns, and to describe these mathematically as relationships or general rules and justify or prove them.

This criterion examines to what extent the student is able to:

- select and apply appropriate inquiry and mathematical problem-solving techniques
- recognize patterns
- describe patterns as relationships or general rules
- draw conclusions consistent with findings
- justify or prove mathematical relationships and general rules.

Assessment tasks for this criterion should be mathematical investigations of some complexity, as appropriate to the level of MYP mathematics. Tasks should allow students to choose their own mathematical techniques to investigate problems, and to reason from the specific to the general. Assessment tasks could have a variety of solutions and may be set in real-life contexts. Teachers should clearly state whether the student has to provide a justification or proof.

Teachers should include a good balance between tasks done under test conditions and tasks done at home in order to ensure the development of independent mathematical thinking.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors given below.
1–2	The student **applies, with some guidance**, mathematical problem-solving techniques to recognize **simple** patterns.
3–4	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, and **suggests** relationships or general rules.
5–6	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, **describes** them as relationships or general rules, and **draws conclusions** consistent with findings.
7–8	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, **describes** them as relationships or general rules, **draws conclusions** consistent with findings, and **provides justifications or proofs**.

Notes

1. Pattern: the underlining order, regularity or predictability between the elements of a mathematical system. To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as relationships or generalized rules.
2. Justification: a clear and logical mathematical explanation of **why** the rule works.
3. Proof: a mathematical demonstration of the truth of a given proposition.

Criterion C: Communication in mathematics

Maximum: 6

Students are expected to use mathematical language when communicating mathematical ideas, reasoning and findings—both orally and in writing.

This criterion examines to what extent the student is able to:

- use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations
- use different forms of mathematical representation (formulae, diagrams, tables, charts, graphs and models)
- communicate a complete and coherent mathematical line of reasoning using different forms of representation when investigating complex problems.

Students are encouraged to choose and use appropriate ICT tools such as graphic display calculators, screenshots, graphing, spreadsheets, databases, drawing and word-processing software, as appropriate, to enhance communication.

Assessment tasks for this criterion are likely to be real-life problems, tests, examinations and investigations. Tests and examinations that are to be assessed against criterion C must be designed to allow students to show complete lines of reasoning using mathematical language.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors given below.
1–2	The student shows **basic** use of mathematical language **and/or** forms of mathematical representation. The lines of reasoning are **difficult to follow**.
3–4	The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **clear** though not always **logical** or **complete**. The student moves between different forms of representation **with some success**.
5–6	The student shows **good** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **concise**, **logical** and **complete**. The student moves **effectively** between different forms of representation.

Notes

1. Mathematical language: the use of notation, symbols, terminology and verbal explanations.
2. Forms of mathematical representation: refers to formulae, diagrams, tables, charts, graphs and models, used to represent mathematical information.

Criterion D: Reflection in mathematics

Maximum: 6

Reflection allows students to reflect upon their methods and findings.

This criterion examines to what extent the student is able to:

- explain whether his or her results make sense in the context of the problem
- explain the importance of his or her findings in connection to real life
- justify the degree of accuracy of his or her results where appropriate
- suggest improvements to the method when necessary.

Assessment tasks are most likely to be investigations and real-life problems. Generally these types of tasks will provide students with opportunities to use mathematical concepts and skills to solve problems in real-life contexts.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors given below.
1–2	The student **attempts** to explain whether his or her results make sense in the context of the problem. The student **attempts to describe** the importance of his or her findings in connection to real life.
3–4	The student **correctly but briefly explains** whether his or her results make sense in the context of the problem and **describes** the importance of his or her findings in connection to real life. The student **attempts** to justify the degree of accuracy of his or her results where appropriate.
5–6	The student **critically explains** whether his or her results make sense in the context of the problem and provides a **detailed explanation** of the importance of his or her findings in connection to real life. The student **justifies** the degree of accuracy of his or her results where appropriate. The student **suggests improvements** to the method when necessary.

Notes

1. Describe: present an account without providing reasons or explanations.
2. Explain: give a detailed account including reasons, causes or justifications. Explanations should answer the questions "why" and "how".

Determining the final grade

This section explains the process by which a student's overall achievement level (in terms of the assessment criteria) is converted to a single grade.

1. Collecting the information

Teachers will use assessment tasks to make judgments of their students' performance against the assessment criteria at intervals during the final year in the subject. Many of the assessment tasks will allow judgments of levels to be made with regard to more than one criterion.

For the purposes of final assessment, teachers **must** ensure that, for each student, they make **several judgments against each criterion**. This can be achieved by using some kinds of assessment task more than once, or by incorporating other types of assessment activity. MYP mathematics has **four** criteria and so **at least eight** judgments (two per criterion) must be made for each student in the final year for the purposes of final assessment. However, as more-complex tasks will allow students to be assessed against several criteria, final assessment may rest on a limited number of tasks.

Important: If more than one teacher is involved in one subject for a single year group, the school must ensure **internal standardization** is used to provide a common system for the application of the assessment criteria to each student. In joint assessment, internal standardization is best achieved by:

- the use of common assessment tasks
- shared assessment between the teachers
- regular contact between the teachers.

In certain schools, students may be grouped according to ability within the same subject (for example, standard and extended levels in mathematics). In such cases, the teachers' final assessment of student performance across all groups or levels must be based on a **consistent application of the assessment criteria to all students**. A different standard should not be applied to different groups or levels.

2. Making a final judgment for each criterion

When the judgments on the various tasks have been made, teachers will be in a position to establish a final profile of achievement for each student by determining the **single most appropriate level for each criterion**. Where the judgments for a criterion differ for specific assessment tasks, the teacher must decide which level best represents the student's final standard of achievement.

Important: Teachers should not average the levels gained in year 5 for any given criterion. Students can develop academically right up to the end of the programme, and teachers must make a professional judgment (that is also supported by work completed) as to which level best corresponds to a student's general level of performance for each of the criteria towards the end of the programme.

3. Determining the final criterion levels total

The final levels for each criterion must then be added together to give a **final criterion levels total** for mathematics for each student. In mathematics, students have the opportunity to gain a maximum level of 8 for criteria A and B, and a maximum level of 6 for criteria C and D. Therefore the maximum final criterion levels total for mathematics will be 28.

The final criterion levels total is the total that will be submitted to the IB via IBIS (IB information system) for those schools that have registered students to receive IB-validated grades.

4. Determining the final grade for mathematics

Grade boundaries must be applied to the criterion levels totals to decide the final grade for each student.

Please see the *MYP coordinator's handbook* for the table of grade boundaries for mathematics.

All MYP subjects receive final grades in the range from 1 (lowest) to 7 (highest) on the IB record of achievement, where students have been registered for IB-validated grades. The general MYP grade descriptors describe the achievement required for the award of the subject grade. After using the conversion table to determine a student's final mathematics grade, teachers should check the general grade descriptor table to ensure that the description equally reflects the student's achievement.

Schools requiring **IB-validated grades** are required to use **only** the published MYP subject-specific criteria and grade boundaries as a basis for the final results that they submit to the IB (both for moderation and as final assessment for certification).

Other schools (those not requiring IB-validated grades) will use the published criteria together with any additional criteria that they have developed independently, and report internally to students and parents. These schools may decide on their own grade boundaries (if using published and additional criteria), or use the boundaries published by the IB.

Mathematics: Moderation

The following details apply **only** to schools that request **IB-validated grades**.

Please ensure that you also refer to the section "Assessment in the MYP".

Purpose of moderation

The external moderation procedure in all MYP subjects and the personal project exists to ensure that students from different schools and different countries receive comparable grades for comparable work, and that the same standards apply from year to year.

All MYP assessment is carried out by the students' own teachers (or by the supervisors in the case of the personal project). The IB moderation procedures ensure that the final tasks set by those teachers are appropriate and that the final judgments made by these teachers all conform to an agreed scale of measurement on common criteria.

To ensure this comparability and conformity, moderation samples submitted to the IB **must** be assessed using the assessment criteria and achievement levels listed in this guide.

The submission date for moderation samples is before the end of a school's academic year. Tasks submitted for moderation are not absolutely final tasks. Schools must continue to make further assessments of students' work after moderation samples have been submitted, as these later tasks will also contribute towards the student's final criterion levels total.

For general information on moderation, please see *MYP: From principles into practice* (August 2008), in the section "Moderation".

Teachers should note that there are three distinct phases to the moderation process.

- Phase 1: Submission of moderation samples
- Phase 2: Submission of criterion levels totals
- Phase 3: Award of MYP grades

Phase 1: Submission of moderation samples

Schools that request IB-validated grades for their students must register these students following the guidelines in the *MYP coordinator's handbook*. This includes students who are only eligible for the record of achievement along with those who are also eligible for the MYP certificate.

Each moderation sample must include **eight folders of students' work** with each folder representing the work of a single student. The selection of student work should be representative of a range of abilities within the final year group, comprising two comparatively good folders, four folders showing average ability and two comparatively weak folders. Only the work of students registered for IB-validated grades should be submitted. If there are fewer than eight students registered, the sample will therefore have fewer than eight folders. In each folder, teachers must include a completed coversheet *Form F3.1*. An additional folder (the background information folder) containing descriptions of the assessment tasks and background information for each task must be supplied.

Since June 2006, schools that have had minimal adjustments to their results over a three-year period have been instructed to send only four folders of student work instead of eight in the relevant subjects. "Minimal adjustments" means differences between teachers' and moderators' totals of within plus or minus 3. This does not mean that there will be no changes to final grades, as some students' totals will still cross grade boundaries even though the differences, and therefore the moderation factors applied, are small. Schools are advised via the moderation reports whether they can send four folders the following year. The situation is monitored annually and applies only to the subjects that have been identified in the moderation reports. For further information, please contact your MYP coordinator.

Prescribed minimum tasks

To meet the required number of judgments **(two)** for each criterion, the following pieces of work **must** be submitted in each folder.

- A **broad-based classroom test/examination** composed of a range of questions and problems, in familiar and unfamiliar situations, covering at least **three** of the branches of the framework for mathematics, and which allows students to reach all levels of achievement. (*Criterion A is strongly recommended as one of the criteria used to assess this task.*)
- A **mathematical investigation**, done under test conditions, where students are given the opportunity to recognize patterns, describe them as relationships or general rules and justify or prove them. (*Criterion B is strongly recommended as one of the criteria used to assess this task.)*
- A **real-life problem** where students are given the opportunity to apply mathematics to a real-life context, reflect upon and evaluate their findings. (*Criterion D is strongly recommended as one of the criteria used to assess this task.*)

For the moderation of extended mathematics, teachers must indicate in the assessment tasks which topics of the extended part of the framework are being assessed.

Should **a fourth task** be necessary to meet the requirements of two judgments against each criterion, one of the following tasks may be included in the moderation sample.

- A second test/examination that does not have to be broad-based
- A second mathematical investigation that does not have to be done under test conditions
- A second real-life problem

For moderation purposes, it is suggested that schools submit a maximum of four tasks.

Characteristics of the prescribed tasks for moderation

Broad-based classroom test/examination

This task should allow students to demonstrate knowledge and understanding of at least three branches of the framework for mathematics and should consist of questions and problems set in both familiar and unfamiliar situations. Papers consisting exclusively of multiple-choice questions and/or questions requiring "true" or "false" answers are not appropriate, as they do not provide students with the opportunity to demonstrate their understanding through the use of mathematical lines of reasoning. Criterion A should be one of the criteria used to assess this task.

Mathematical investigation

This task should develop from an initial problem that does not have an obvious solution or approach.

The investigation:

- should provide a challenge and an opportunity for creativity
- should allow students to choose different courses of action from a range of options
- may have a variety of answers
- should develop the skills of:
  - producing a strategy
  - generating data
  - recognizing patterns or structures
  - searching for further cases
  - formulating, testing and justifying or proving a general rule.

Mathematical investigations should be designed and carried out independently by the student. Teacher-guided tasks where students simply follow a procedure are not suitable tasks.

Criterion B should be one of the criteria used to assess this task.

Real-life problem

This task could be inspired by the relationship between mathematics and other areas of knowledge such as the sciences, the physical world, the environment, the economy, technology, health, medicine or society. The task should also highlight the role of mathematics in the real world.

This task should allow the student to apply mathematics to real life by:

- identifying the problem
- translating the problem into mathematics
- solving the problem
- interpreting the solutions in the real-life context.

Criterion D should be one of the criteria used to assess this task.

Important notes

- In the moderation sample, teachers' assessments of students' work must be based entirely on the criteria published in this guide.
- Each criterion (A, B, C, D) must have **two** judgments made against it for the purposes of moderation.
- Apart from the criteria suggested for the prescribed tasks above, teachers may use other criteria, as appropriate, to assess student work.
- Teachers should ensure that the correct number of judgments is recorded for each criterion on the coversheet *Form F3.1*. The reverse of coversheet *Form F3.1* should contain information on extenuating circumstances for individual students, if this is not already included in the background information.
- Tasks for moderation must be devised to give students the opportunity to reach the highest level of each criterion.
- Background information should document details that may be useful to the moderators. This should include descriptions of the assessment tasks, time allocation, the degree of teacher support, and the conditions under which the task was completed. Teachers should also supply evidence illustrating the application of the assessment criteria (for example, markschemes, a copy of the relevant worksheet or test paper), comments on student work, and descriptions of the ways the assessment tasks were presented to the students.
- Background information in mathematics should also indicate which concepts and skills were the specific focus of the assessment tasks.
- If teachers use third-party material as stimuli and/or as part of their tasks, this material must be fully referenced. This will include the title of the source, the author, the publication date, the publisher and, for books only, the ISBN. Examples of third-party material include newspaper and magazine articles, cartoons, videos, movie excerpts, extracts from books, pictures (please check the acknowledgments in the original publication for the original sources), diagrams, graphs, tables, statistics, materials from websites, and so on.
- Background information should be compiled into a ninth folder. This information does not need to be added into each of the eight student folders. The background information must be submitted in the working language of the school (English, French, Spanish or Chinese).
- Student work submitted for moderation should reflect the types of assessment tasks used later in the year by the teacher for final assessment.
- Where possible, original student work should be submitted rather than photocopies, though schools are encouraged to keep photocopies of the work in school.
- Although group work is encouraged in practice, group work must not be submitted for moderation purposes. It is difficult for moderators to ascertain a student's actual contribution to a piece of work that was undertaken in a group situation.
- Teachers may have devised tasks to focus on only one or two of the criteria, and therefore do not make two judgments against each criterion with the three prescribed minimum tasks. In this case it is acceptable to enclose additional tasks in the sample until each criterion has been assessed twice. If a criterion has been assessed more than twice in the sample, the extra assessment(s) will not be moderated.
- Anything in the moderation sample that differs from the stated requirements should be explained in the background information.

The *MYP coordinator's handbook* provides the coversheet *Form F3.1*, and further guidance on submitting moderation samples in each subject. The mathematics teacher support material provides examples of key components of a moderation sample.

Phase 2: Submission of criterion levels totals

Phase 1 of the moderation process takes place before the end of most schools' academic year. After submitting moderation samples, teachers should continue to assess students' work until **final assessment**.

After final assessment, teachers should use the procedure described in "Determining the final grade" to arrive at a **criterion levels total** for each student registered for certification.

The MYP coordinator will then enter each registered student's criterion levels total on **IBIS** (IB information system), and submit this to the IB.

Phase 3: Award of MYP grades

Following moderation in each subject, the IB may, where appropriate, apply a moderation factor to the criterion levels totals submitted by a school. Final grades will then be determined by applying grade boundaries to these moderated totals.

Schools will receive notification of the final grades for their students and the IB will also provide a general and a school-specific moderation report for each subject in which students were registered.

The *MYP coordinator's handbook* provides further guidelines on submitting criterion levels totals in each subject.

Mathematics: Monitoring of assessment

The following applies to schools **not** requesting IB-validated grades.

Please ensure that you also refer to the sections "Assessment in the MYP" and "Mathematics: Moderation".

Definition

Monitoring of assessment is a service available to IB World Schools offering the MYP, whereby schools can send samples of assessed student work to the IB to receive feedback from an experienced MYP moderator in the form of a report. This service is subject to a fee.

Monitoring of assessment is aimed at providing support and guidance in the implementation and development of the programme with regard to internal assessment procedures and practices. It is not linked to validation of students' grades, and therefore differs from the process of external moderation. Monitoring of assessment is currently limited to assessment conducted in the final three years of the programme.

Samples for monitoring of assessment in mathematics must be submitted in English, French, Spanish or Chinese, although these may be translations into one of these languages.

Details on registering for monitoring of assessment and fees, as well as the latest updated versions of the coversheets, are available in the *MYP coordinator's handbook*. Examples of completed coversheet *Form F4.4* are available in the *Mathematics teacher support material*.

Further information on monitoring of assessment can be found in the document *MYP: From principles into practice* (August 2008), in the section "Monitoring of assessment". Brief information follows here.

Purpose

There are three reasons why schools send in a monitoring of assessment sample:

1. as a requirement for the school's programme evaluation visit
2. as a pre-check before sending in samples for moderation
3. to receive guidance on a particular subject.

Choice of tasks for monitoring of assessment

For evaluation visit and general advice

Schools can decide on the types of task they wish to submit for monitoring of assessment for the evaluation visit or for general advice. However, they are recommended to consider the prescribed minimum tasks detailed in the "Mathematics: Moderation" section, as these are designed to give an even spread over the mathematics assessment criteria (A, B, C and D).

Prior to moderation

If the school is requesting monitoring of assessment in preparation for future moderation, the tasks in the following list **must** be included in the sample of assessed student work. These are the required minimum tasks listed in the "Mathematics: Moderation" section.

- A **broad-based classroom test/examination** composed of a range of questions and problems, in familiar and unfamiliar situations, covering at least **three** of the branches of the framework for mathematics, and which allows students to reach all levels of achievement. (*Criterion A is strongly recommended as one of the criteria used to assess this task.*)
- A **mathematical investigation**, done under test conditions, where students are given the opportunity to recognize patterns, describe them as relationships or general rules and justify or prove them. (*Criterion B is strongly recommended as one of the criteria used to assess this task.)*
- A **real-life problem** where students are given the opportunity to apply mathematics to a real-life context, reflect upon and evaluate their findings. (*Criterion D is strongly recommended as one of the criteria used to assess this task.*)

Please see the "Mathematics: Moderation" section for further notes and information.

MYP mathematics frequently asked questions

General

How is MYP mathematics different from other mathematics courses?

MYP mathematics aims to provide students with the mathematical competences and the intellectual capacity for pursuing further studies in mathematics and for becoming confident users of mathematics later in life. Therefore, MYP mathematics should be accessible to and be studied by all students.

Unlike other mathematics courses, where the emphasis is placed solely on developing mathematical knowledge and skills, MYP mathematics focuses on the development of an intellectual capacity for lifelong learning. The final objectives for MYP mathematics support the IB learner profile by promoting the development of students who are knowledgeable, inquirers, communicators and reflective thinkers.

MYP mathematics encourages the use of investigations as a means to support inquiry-based learning, while allowing students to explore concepts and solve problems using mathematical skills and reasoning. Communication and reflection are fundamental to the teaching and learning of MYP mathematics, and contribute to the development of analytical and critical thinkers who can access information and communicate ideas and findings confidently using the language of mathematics.

MYP mathematics encourages teachers to use problems and investigations set in real-life contexts, to help students see the connections between mathematics and other areas of life, and to appreciate the role of mathematics in life and society.

How does the MYP prepare students for the Diploma Programme?

MYP mathematics aims to prepare students for further post-16 courses in mathematics, including the mathematics standard level (SL) and mathematics higher level (HL) courses of the Diploma Programme (DP).

The MYP framework for mathematics was devised to provide sufficient breadth and depth to meet the needs of students wishing to study mathematics at DP level.

The framework for mathematics was revised in conjunction with the DP curriculum to cover the concepts and skills of the presumed knowledge (PK) for courses at mathematics SL and HL. The two levels of the MYP framework (standard and extended) were refined to allow a smooth transition from MYP mathematics to DP mathematics courses.

Can I teach to objectives other than those listed in the *Mathematics guide*?

Teachers may teach to objectives in addition to those listed. However, students must be given the opportunity to achieve all of the objectives listed in this guide by the end of the final year of the MYP.

What level of mathematics should we offer? Do we need to separate students by ability?

The MYP framework for mathematics allows students to work at two levels: standard and extended mathematics. However, it is the prerogative of the school to decide whether to offer one or both levels, and whether to group students by ability or to teach mixed-ability classes.

These decisions should be based on the analysis of a number of factors, including local and national requirements, the characteristics of the student population, and the availability of resources in the school.

The majority of schools offering both standard mathematics and extended mathematics do not teach students of different ability in different classes until they are in MYP year 3.

Students aspiring to study mathematics at higher level (HL) as part of the DP would be best advised to study MYP extended mathematics. However, many schools prepare students for DP mathematics HL within the MYP standard mathematics course.

Should schools address every branch of the framework in every year of the programme?

No, it is not a requirement for schools to address all the branches of the framework in each year of the programme. Nor are schools expected to cover every concept and skill of the five branches in their school syllabus, as if they were ticking boxes.

Schools should decide how to structure and sequence their instruction, using the framework as a guideline, so that the courses they design best address the needs of their students while also complying with local and national requirements. For example, schools may want to place a stronger emphasis on some branches in the early years and leave the others for later exploration, or they may prefer to address some concepts and skills across all branches as students progress through the MYP.

However, in order to provide students with opportunities to meet the final objectives of MYP mathematics, schools must ensure that, over the five years (or complete duration) of the programme, students experience learning in number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics.

How much of the framework for mathematics needs to be covered at the different levels?

The framework for mathematics should not be confused with a syllabus where teachers are expected to cover all the topics in a course. The framework should act as a guide to help teachers design and plan their courses at both levels. The framework for mathematics is structured into five branches, listing suggested concepts and skills for each branch for both standard and extended levels.

The mathematics curriculum that the school designs must allow students to experience learning in all five branches of the framework by the end of the programme. Schools must also ensure that the courses developed do not compromise the final objectives of MYP mathematics.

How can I help students who have difficulties with the level of standard mathematics?

Some students may need additional support to meet the objectives of MYP standard mathematics. In this case it is important to determine students' needs in order to best support their learning. Students may be accessing the course in a language other than their mother tongue, or may have a diagnosed or non-diagnosed special educational need (SEN). Any of these cases needs to be investigated and identified so that schools can organize appropriate support systems for these students. This may include teacher training, language support courses, subject support tutorials, peer mentoring, differentiated instruction, modification of assessment tasks, or an interview with an SEN specialist to make a diagnosis and guide further instruction.

For further information on these issues, please refer to the document *Learning in a language other than mother tongue in IB programmes* and to the SEN page, resources and forums on the online curriculum centre (OCC).

How can I adapt mathematics teaching to meet the needs of students with special educational needs?

There are a number of ways in which teachers can differentiate their instruction in order to meet students' special educational needs in mathematics. Teachers should focus on the development of basic skills and competences to support students' mathematical ability as part of lifelong learning skills for the workplace.

Some suggestions for differentiated instruction in mathematics are listed below.

- Use real-life contexts for the application of knowledge and skills when devising problem-solving situations, in order to provide meaningful experiences for students.
- Differentiate tasks to incorporate the wide range of students' skills and capabilities.
- Use direct instruction when teaching basic skills, in order to help students process the vocabulary, text format, structure and use of symbols in mathematics.
- Encourage cooperative learning strategies and an individualized approach.
- Encourage repetition and allow additional time for consolidation and reinforcement of knowledge.
- Limit the number of exercises and concentrate on enhancing understanding. Do not expect students with SEN to do as many problems as others; limit the number required to show understanding and application.
- Use ICT (for example, calculators, mathematical software) to aid and support understanding.
- Develop a tiered approach to knowledge to suit different students' needs:
  - core (basic) knowledge that has to be known by all
  - standard knowledge that should be known by most
  - more challenging and "interest-based" knowledge for those who wish to stretch themselves.
- Promote the design of projects that offer choices related to the students' interests (for example, analysing costs of various mobile phone providers).
- Monitor and record progress on an ongoing basis and devise yearly plans targeting different groups of students.

For more strategies please refer to the SEN page, resources and forums on the OCC.

How can I detect plagiarism? How can I avoid it in the first place?

If you suspect that work has been plagiarized, one way to check is to conduct an Internet search. Using a major search engine, type in a selection of the work in inverted commas (one sentence should be sufficient). If the work has been taken directly from a website it will be detected. Your school may also subscribe to a plagiarism detection site. Plagiarism from other sources can be more difficult to detect, depending on how familiar the teacher is with all the resources available to the students.

The best solution is to avoid setting tasks that are easy to complete through plagiarism, or other forms of academic dishonesty. Tasks should be challenging, but not so difficult that students are tempted to use dishonest means to complete them, and support should be available when students require it. For further information on IB policy on academic honesty, please see the publication *Academic honesty*.

Does the IB recommend any particular style of referencing/quoting/footnoting?

There is no set style for referencing in the MYP. Schools need to decide on one or more recognized styles of referencing that suits the needs of the students and the school.

Can we use teaching resources if we do not have a clear idea of where the resources came from?

Teachers need to adhere to the guidelines of academic honesty as much as the students. Therefore teachers need to make every effort to reference and acknowledge the work of others that they use in the classroom.

Why are the overall grade boundaries not included in the guide?

The grade boundaries are included in the *MYP coordinator's handbook* (available on the OCC), which is updated every year. This gives the flexibility to adjust grade boundaries if necessary after the first moderation session.

Assessment

I want to assess my students in a wide variety of ways without being restricted to the choice of a "test/examination", a "mathematical investigation" or a "real-life problem". Can I assess in other ways as well?

Yes. The "test/examination", "mathematical investigation" and "real-life problem" are the required minimum tasks for moderation/monitoring of assessment, and are only a snapshot of what is assessed in schools. Student ability should be assessed through a wide range of assessment activities during all years of MYP instruction.

Why change the investigation to "test conditions"?

The reasons for this decision were twofold. Firstly, to reduce the overall workload for students, and secondly, to ensure that authentic and individual work is submitted for moderation. Setting the investigation under test conditions provides time constraints and might limit students' scope for inquiry. Therefore, teachers should ensure that the tasks they design provide sufficient information for students to investigate and allow them enough time to complete their investigation during the time set for the test.

What is an open-ended investigation in mathematics?

An open-ended investigation is one that has more than one possible solution, or more than one method of approach. An example of the former could be: "Design a gambling game that would allow a casino to win 51% of the time." An example of the latter could be: "From the past 100 years of records in the 1,500 metre run, predict what the record will be in 2050."

What is the difference between "apply" and "select and apply" in the context of the wording of criterion B?

In level 1–2 students apply a mathematical problem-solving technique with some guidance. In this context "apply" implies that the student may have received some help in the form of instructions for the problem or from the teacher.

However, in order to achieve level 3–4 students are expected to "select and apply" mathematical problem-solving techniques without guidance. "Select" in this context implies that the student has considered different alternatives and has consciously and independently decided which mathematical technique to use.

What is the connection between the criterion levels and the final grade?

A criterion level only gives a partial assessment of mathematics. For example, a level for criterion C only shows a student's achievement in "communication in mathematics", and does not give an overall picture of his or her mathematical competences. To work out a student's final grade, a teacher must take into account levels from all of the criteria, giving a balanced final result. In summary, the final grade is an overall view of the student's achievement in the subject; the criterion levels show a student's achievement in components of the subject.

For example:

	Criterion A (/8)	Criterion B (/8)	Criterion C (/6)	Criterion D (/6)	Levels total (/28)	Final grade
Student 1	5	4	6	6	21	5
Student 2	8	8	3	3	22	5

Criterion levels and final grades are useful in different ways. For example, schools may use final grades for reporting to parents, but use criterion levels in designing their lessons, as these give more specific feedback on the needs of the students.

Moderation

What are the requirements for moderation of extended mathematics?

Extended mathematics has the same moderation requirements as standard mathematics. Both have to meet the prescribed minimum assessment tasks as listed in the "Mathematics: Moderation" section of this guide. However, for those tasks submitted **for extended mathematics** teachers must clearly indicate in the tasks which topics of the extended framework have been assessed.

How can I address each criterion twice with the three required tasks for moderation?

When designing a task, teachers should check the criteria descriptors to ensure that the task is suitable for assessment against the desired criteria and that it allows students the opportunity to reach the highest achievement levels.

Although, in theory, any type of task could be made appropriate for any of the four criteria, a broad-based test would be most appropriate for the assessment against criterion A, a mathematical investigation for the assessment against criterion B, and a real-life problem for the assessment against criterion D, as shown in the table below.

Task	A	B	C	D
Broad-based test	✓			
Mathematical investigation		✓		
Real-life problem				✓

In order to provide two judgments per criterion using the minimum number of assessment tasks, teachers should explore how to adapt the tasks so that they can be assessed against more than one criterion.

A test can generally be written in such a way that it can also be assessed against communication (criterion C), but it would be difficult to assess a test against criteria B and D reliably. A mathematical investigation could be designed to be assessed against criteria B, C and D, while a real-life problem could be written so that it is also suitable for assessment against criteria A and C.

Task	A	B	C	D
Broad-based test	✓		(✓)	
Mathematical investigation		✓	(✓)	(✓)
Real-life problem	(✓)		(✓)	✓

✓ suggested
(✓) could possibly be assessed

If two judgments against each criterion cannot be made sensibly using three assessment tasks, a fourth one may be submitted.

What is "background information"? What should I include?

Background information is the information provided in a moderation or monitoring of assessment sample that tells the moderator or assessor details of the tasks, what the expectations were, what resources were available and under what conditions the tasks were completed. Examples of background information include worksheets, instructions or notes given to students, information on time allocation/length of preparation, degree of teacher or peer support allowed, blank copies of tasks, and comments on student

work. In mathematics it is important that background information indicates the degree of assistance the students received when working on the tasks for moderation. Responses to suggestions made in the previous moderation report, or monitoring of assessment, must be included whenever relevant.

If teachers use third-party material as stimuli and/or as part of their tasks, this material must be fully referenced. This will include the title of the source, the author, the publication date, the publisher and, for books only, the ISBN. Examples of third-party material include newspaper and magazine articles, cartoons, videos, movie excerpts, extracts from books, pictures (please check the acknowledgments in the publication for the original sources), diagrams, graphs, tables, statistics, materials from websites, and so on.

If the sample differs from the stated requirements in any way, this should also be explained in the background information.

If the tasks I give students are not appropriate, will my students be penalized?

If the tasks submitted for moderation do not give the students the opportunity to demonstrate all of the skills listed in the corresponding criterion/criteria, or if the standard of performance expected is too low for final-year students, then the task will be deemed inappropriate.

When tasks are inappropriate, it is often the case that the levels awarded by the teacher are too high. In these cases, the levels will be lowered appropriately, and this may result in students' final grades also being lowered.

Do all the tasks that we send for moderation have to involve ICT, and third-party media/resources?

No, this is not a moderation requirement. ICT should be used as and when appropriate to support learning, enhance understanding or make communication more effective. Naturally, if third-party resources are used in tasks, students and teachers are expected to acknowledge the sources according to a recognized convention.

MYP mathematics glossary

Context	The situation and the parameters given to a problem.
Deductive reasoning	Reasoning from the more general to the specific.
Direct instruction (SEN)	Teaching approach where the teacher will transmit information directly to the students, generally in a very structured way (step-by-step approach). The teacher provides immediate feedback, modelling situations and allowing time for independent practice. Direct instruction is particularly effective for teaching basic skills.
Forms of (mathematical) representation	Refers to formulae, diagrams, tables, charts, graphs and models used to represent mathematical information.
General rules	These include formulae, theorems, proven relationships and laws.
Inductive reasoning	Involves making generalizations from specific observations, recognizing patterns and making general rules.
Justification	A clear and logical mathematical explanation of **why** a rule works.
Mathematical investigation	An investigation that requires students to reason from the specific to the general; may have a variety of solutions and may be set in a real-life situation.
Mathematical language	The use of notation, symbols, terminology and verbal explanations.
Mixed-ability classroom	Students with all abilities share the same classroom; no grouping or teaching differentiation is applied.
Pattern	The underlying order, regularity or predictability between the elements of a mathematical system.
Proof	A mathematical demonstration of the truth of a given proposition.
Unfamiliar situation	This refers to challenging questions or instructions set in a new context in which students are required to apply knowledge and/or skills they have been taught.